Microcavity polaritons in disordered exciton lattices

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We investigate the interaction of excitons in a two-dimensional lattice and photons in a planar cavity in the presence of disorder. The strong exciton-photon coupling is described in terms of polariton quasiparticles, which are scattered by a disorder potential. We consider three kinds of disorder: (i) inhomogeneous exciton energy, (ii) inhomogeneous exciton-photon coupling, and (iii) deviations from an ideal lattice. These three types of disorder are characteristic of different physical systems. Their separate analysis gives insight into the competition between randomness and light matter coupling. We consider conventional planar polariton structures (in which excitons are resonant with photon modes emitting in the direction normal to the cavity plane) and Bragg polariton structures (in which excitons in a lattice are resonant with photon modes at a finite angle satisfying the Bragg condition). We calculate the absorption spectra in the normal direction and at the Bragg angle by direct diagonalization of the exciton-photon Hamiltonian. We found that in some cases weak disorder increases the light matter coupling and leads to a larger polariton splitting. Moreover, the coupling of excitons and photons is less sensitive to disorder of type (ii) and (iii). This suggests that polaritonic structures realized with impurities in a semiconductor or with atoms in an optical lattice are a good candidate for the observation of some of the Bragg polariton features.

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I. INTRODUCTION

There is currently a considerable interest in the physics of strongly coupled light matter systems. A particular case is given by experiments on the Bose-Einstein condensation (BEC) of microcavity polaritons,[1–4] which are mixed states of excitons and photons (see Refs. 5 and 6). In order to combine the properties of matter states in a lattice and planar photon modes, we have recently investigated a structure consisting of an array of quantum dots in a planar cavity,[7] as schematically shown in Fig. 1. This particular geometry for light matter coupling can be realized in many different ways using either semiconductor based systems (quantum dots, impurities, and metallic gates) or atomic systems such as atoms in optical lattices. Experimental and theoretical investigations show that impurity bound excitons can have a very small inhomogeneous broadening and a strong confinement energy [e.g., of the order of 50 meV (Refs. 8 and 9)]. Moreover, ordered arrays of single dopants,[10] as well as the control of a single impurity using scanning tunneling microscopy, have been experimentally demonstrated.[11,12]

Polaritons in structures similar to the one in Fig. 1 but involving photonic crystals[13] or optical lattices[14,15] have been recently investigated theoretically. Many of these structures have planar spatial periodicity, which leads to very interesting properties. We have recently theoretically explored Bragg polariton modes at specific symmetry points of the Brillouin zone boundaries.[7] These zone edge Bragg polaritons can have extremely small effective masses, typically three orders of magnitude smaller than conventional cavity polaritons, and behave effectively as Dirac quasiparticles. Bragg polaritons can therefore be similar to light-mass (relativistic) electrons in graphene, which have been extensively investigated recently.[16,17] Polaritons with Dirac dispersion could have interesting applications to the physics of polariton BEC and superfluidity, as well as to spin coupling control.[18]

In Fig. 1 we show schematically disorder effects due to fluctuations in the quantum dot size and position. These disorder effects are the main focus of the present paper. The effect of disorder on polaritons has been extensively studied in the quantum well microcavity case (see Ref. 5 and references therein). It was found that if the potential fluctuations due to the disorder are comparable to the Rabi splitting then the two polaritonic peaks disappear and one inhomogeneously broadened peak remains in the absorption spectrum. Furthermore, a similar behavior has been predicted for an ensemble of two level systems coupled to a single-photon mode.[19] The dynamics of the wave packet and its localization in a disordered one-dimensional cavity have been studied as well.[20] In this paper, we investigate the effect of energy, oscillator strength, and position fluctuations on polariton modes. Disorder effects could also originate from the mirrors, for instance due to fluctuations in the growth of the distributed Bragg reflectors (DBR). Here we consider perfect mirrors. Numerical results are discussed and compared with a perturbation theory approach. The paper is organized as follows: The theory is introduced in Sec. II and is followed by results and discussion in Sec. III. Conclusions are drawn in Sec. IV.

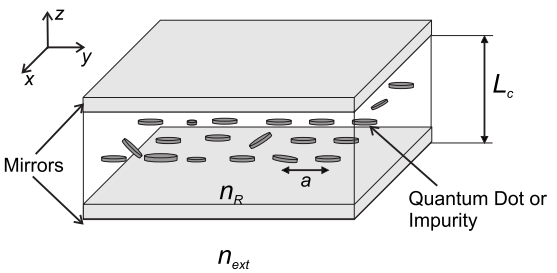

FIG. 1. Scheme of the investigated system. A lattice of quantum dots or impurities is embedded in a planar microcavity. Energy, position, and oscillator strength of the localized excitons fluctuates from site to site.

II. THEORY

We are going to investigate a disordered planar lattice embedded in a planar microcavity structure. Excitons are localized in a lattice but the physical mechanism of localization will not be specified at this point. As discussed above, quantum dots, impurities, metal gates, and optical lattices could be used to localize exciton states. We further assume that only one excitonic level is present on each site, implying a strong localization. The case of quantum dots has been extensively discussed in Ref. 7. We start with the Hamiltonian in second quantization ($\hbar=1$),

$$\hat{H}=\sum_j \omega_j C_j^\dagger C_j+\sum_q \omega_q a_q^\dagger a_q+\sum_{jq}(ig_{jq}e^{iqR_j}a_q^\dagger C_j+\text{H.c.}), \tag{1}$$

where $C_j^\dagger$ is the exciton creation operator on the jth site at position R_j with energy ω_j and exciton-photon coupling g_{jq} and $a_q^\dagger$ is the creation operator of the cavity photon mode with in-plane momentum q and energy ω_q. By writing the coupling constant as $g_{jq}e^{iqR_j}$, we can analyze separately the disorder effect induced by position fluctuations and oscillator strength fluctuations.

A. Disorder properties

We assume that the distance between localization sites is much larger than the effective exciton localization length on each site. Therefore, fluctuations on different sites are uncorrelated. For energy fluctuations $\Delta\omega_j=\omega_j-\omega_X$ (ω_X is the average exciton energy), we have

$$\langle\Delta\omega_j\rangle=0, \tag{2}$$

and

$$\langle\Delta\omega_m\Delta\omega_n\rangle=\sigma_\omega^2\delta_{mn}, \tag{3}$$

where $\langle.\rangle$ indicates averaging over a Gaussian ensemble. Similarly, for the oscillator strength disorder we have

$$\langle g_{jq}\rangle=\langle g_{0q}\rangle=\overline{g_{0q}},\quad \langle g_{kq}g_{lq'}\rangle-\langle g_{kq}\rangle\langle g_{lq'}\rangle=\sigma_{qq'}^2\delta_{kl}, \tag{4}$$

where $g_{0q}=ge^{-q^2\beta^2/4}$, β is the characteristic exciton localization length,[7] and

$$\sigma_{qq'}^2=\sigma_g^2e^{-(q^2+q'^2)\beta^2/4}. \tag{5}$$

In Eqs. (4) and (5), $\sigma_g^2=\langle g^2\rangle-\langle g\rangle^2$ is the on-site oscillator strength variance at $q=0$. Another possibility to model this type of disorder is to assume fluctuations in β, which would be appropriate for the quantum dot case but not for impurities with Frenkel-like exciton. Finally, for the positional disorder we obtain

$$\langle R_j\rangle=\overline{R_j},\quad \langle R_kR_l\rangle-\langle R_k\rangle\langle R_l\rangle=\sigma_R^2\delta_{kl},$$

$$\langle e^{iqR_j}\rangle=e^{iq\overline{R}_j}e^{-q^2\sigma_R^2/2}, \tag{6}$$

where the positions $\overline{R_j}$ identify the ideal two-dimensional lattice. Furthermore, we assume that the different kinds of disorder are uncorrelated. This is not fully justified in the case of quantum dots but it allows us to treat the different types of disorder using the perturbation theory and to get insight into their different properties.

B. Polariton scattering

We can separate the disorder-free part of the total Hamiltonian by defining the Fourier transform of the exciton operators as

$$C_j^\dagger=\frac{1}{\sqrt{N}}\sum_{q\in 1.BZ}e^{iq\overline{R}_j}C_q^\dagger,$$

$$C_j=\frac{1}{\sqrt{N}}\sum_{q\in 1.BZ}e^{-iq\overline{R}_j}C_q, \tag{7}$$

where $\overline{R_j}$ is the jth site of a two-dimensional ideal lattice and N is the total number of sites. The sum over the q states is restricted to the first Brillouin Zone (BZ) of the reciprocal lattice space. The disorder-free Hamiltonian then takes the form of

$$\hat{H}_0=\sum_q\left\{\omega_XC_q^\dagger C_q+\sum_Q\omega_{q+Q}a_{q+Q}^\dagger a_{q+Q}+\sum_Q(\tilde{g}_{q+Q}a_{q+Q}^\dagger C_q+\text{H.c.})\right\}, \tag{8}$$

where Q is a reciprocal lattice vector and $\tilde{g}_q=\sqrt{N}\,g_{0q}$ is the renormalized coupling constant. We can then define the disorder coupling constants,

$$\Delta\omega_q=\frac{1}{\sqrt{N}}\sum_j e^{iq\overline{R}_j}\Delta\omega_j,$$

$$\eta_{qq'}^o=\frac{1}{\sqrt{N}}\sum_j e^{i(q-q')\overline{R}_j}(g_{jq}-\bar{g}_{0q}),$$

$$\eta_{qq'}^p=\frac{1}{\sqrt{N}}\sum_j(e^{iqR_j}-e^{iq\overline{R}_j})e^{-iq'\overline{R}_j}\bar{g}_{0q}, \tag{9}$$

where the index o (p) indicates the oscillator strength (positional) disorder. The statistical properties of these functions are determined by their average values as

$$\langle\Delta\omega_k\rangle=0,$$

$$\langle\eta_{qq'}^o\rangle=0,$$

$$\langle\eta_{qq'}^p\rangle=\sum_K\delta_{q,q'+K}\xi_q, \tag{10}$$

and correlations according to

$$\langle\Delta\omega_k\Delta\omega_l^*\rangle=\sigma_\omega^2\delta_{kl},$$

$$\langle\eta_{qq'}^o\eta_{kk'}^{*o}\rangle=\sum_K\delta_{q-k,q'-k'+K}\sigma_{qk},$$

$$\langle \eta^{p}_{qq'} \eta^{*p}_{kk'} \rangle - \langle \eta^{p}_{qq'} \rangle \langle \eta^{*p}_{kk'} \rangle = \sum_{K} \delta_{q-k,q'-k'+K} \zeta_{qk}, \quad (11)$$

where K is a reciprocal lattice vector, and

$$\xi_q = \tilde{g}_{0q}(e^{-q^2\sigma_R^2/2} - 1),$$

$$\zeta_{qk} = \tilde{g}_{0q}\tilde{g}^{*}_{0k}(e^{-|q+k|^2\sigma_R^2/2} - e^{-q^2\sigma_R^2/2}e^{-k^2\sigma_R^2/2}). \quad (12)$$

The disorder terms of the Hamiltonian for the three different mechanisms can then be written in a compact form as

$$\hat{H}^{e} = \frac{1}{\sqrt{N}} \sum_{qk} \Delta\omega_k C^{\dagger}_{q+k} C_q,$$

$$\hat{H}^{o(p)} = \sum_{qq'} (\eta^{o(p)}_{qq'} a^{\dagger}_{q} C_{q'} + \text{H.c.}). \quad (13)$$

We can find the eigenvalues and eigenvectors of the disorder-free Hamiltonian by solving the problem,

$$\hat{H}_0 |P_{nq}\rangle = \Lambda_{nq} |P_{nq}\rangle. \quad (14)$$

The corresponding eigenstates are the disorder-free polariton states, which can be written in the form,

$$|P_{nq}\rangle = P^{\dagger}_{nq}|0\rangle = \left(u_{nq} C^{\dagger}_{q} + \sum_{Q} v_{nq+Q} a^{\dagger}_{q+Q}\right)|0\rangle, \quad (15)$$

where n is a band index, v_{nq} and u_{nq} are Hopfield coefficients,[21] and $|0\rangle$ is the exciton-photon vacuum. We can use these states and obtain an effective disorder potential for polaritons as

$$V^{J}_{nn'qq'} = \langle P_{nq}|\hat{H}_{1(2)}|P_{n'q'}\rangle,$$

where $J \in \{e, o, p\}$ labels the energy, oscillator strength, and positional disorder, respectively. This gives for the energy disorder,

$$V^{e}_{nn'qq'} = \frac{\omega_{q-q'}}{\sqrt{N}} u^{*}_{nq} u_{n'q'}, \quad (16)$$

and for the oscillator strength (positional) disorder,

$$V^{o(p)}_{nn'qq'} = \sum_{Q} [\eta^{o(p)}_{q+Qq'} u_{n'q'} v^{*}_{nq+Q} + \eta^{*o(p)}_{q+Qq'} u^{*}_{nq} v^{*}_{n'q'+Q}]. \quad (17)$$

C. Absorption spectrum

The absorption spectrum at an angle determined by the in plane q of the cavity photon can be calculated using the full propagator of the disordered system, which can be written in terms of the disorder-free polariton states as

$$\hat{G}(\omega) = \sum_{nn'kk'} |P_{nk}\rangle G_{nn'kk'}(\omega) \langle P_{n'k'}|, \quad (18)$$

with

$$G_{nn'kk'}(\omega) = G^{0}_{nk}(\omega)\delta_{nn'}\delta_{kk'} + G^{0}_{nk}(\omega) T_{nn'kk'}(\omega) G^{0}_{n'k'}(\omega), \quad (19)$$

where

$$G^{0}_{nk}(\omega) = \frac{1}{\omega - \Lambda_{nk} + i\epsilon} \quad (20)$$

is the Green's function for disorder-free polaritons. T is the scattering T matrix that can be expressed as

$$T(\omega) = \sum_{j} T^{(j)}(\omega),$$

$$T^{(j)}_{nn'kk'}(\omega) = \langle P_{nk}|\hat{V}(\hat{G}^{0}(\omega)\hat{V})^{j}|P_{n'k'}\rangle. \quad (21)$$

The imaginary part of the polariton propagator projected on a photon mode at a given wave vector $|\gamma\rangle \equiv |k_\gamma\rangle$ gives the absorption spectrum at the corresponding excitation angle as

$$A_\gamma(\omega) = -\,\text{Im}\, G_\gamma(\omega),$$

with

$$G_\gamma(\omega) = \langle \gamma|\hat{G}(\omega)|\gamma\rangle = \sum_{nn'} v_{nk_\gamma} v^{*}_{n'k_\gamma} G_{nn'k_\gamma k_\gamma}(\omega). \quad (22)$$

We can explicitly average over the disorder configurations to obtain

$$\langle G_\gamma(\omega)\rangle = \sum_{nn'} v_{nk_\gamma} v^{*}_{n'k_\gamma} \langle G_{nn'k_\gamma k_\gamma}(\omega)\rangle,$$

with

$$\langle G_{nn'k_\gamma k_\gamma}(\omega)\rangle = G^{0}_{nk}(\omega)\delta_{nn'}\delta_{kk'} + G^{0}_{nk_\gamma}(\omega) \times \langle T_{nn'k_\gamma k_\gamma}(\omega)\rangle G^{0}_{n'k_\gamma}(\omega). \quad (23)$$

In the case of disorder caused by energy inhomogeneity, the ensemble averaging leads to the momentum conservation $[\langle T_{nn'kk'}(\omega)\rangle = \langle T_{nn'kk}(\omega)\rangle \delta_{kk'}]$ since by averaging only the diagonal terms in Eq. (11) remain. We note that this is similar to the problem of a single-particle scattering in a random ensemble of impurities.[22] The diagonal elements of the T matrix can be interpreted as an effective self-energy, $\Sigma_{nk}(\omega) = T_{nnkk}(\omega)$, and within the pole approximation $\Sigma_{nk}(\omega = \Lambda_{nk})$ their real part gives an energy shift of the polariton levels. This shift can also be calculated using a perturbation theory approach as shown in the following section.

D. Perturbation theory

Here we calculate the corrections to the energy up to the second order in the disordered potential using the statistical properties of the disorder derived above. We start with the first-order energy shift for all three kinds of disorder;

$$E^{1,e}_{nk} = \frac{\Delta\omega_0}{\sqrt{N}} |u_{nk}|^2, \quad E^{1,o(p)}_{nk} = 2\ \text{Re}\left\{\sum_{K} \eta^{o(p)}_{k+Kk} u_{nk} v^{*}_{nk+K}\right\}, \quad (24)$$

where u_{nk} and v^{*}_{nk+K} are defined in Eq. (13). The average value of the energy shift,

$$\langle E_{nk}^{1,e}\rangle = 0, \tag{25}$$

$$\langle E_{nk}^{1,o}\rangle = 0, \tag{26}$$

$$\langle E_{nk}^{1,p}\rangle = 2\ \mathrm{Re}\left\{\sum_K \xi_{k+K} u_{nk} v_{nk+K}^*\right\}, \tag{27}$$

and variances are

$$\langle (E_{nk}^{1,e})^2\rangle = \frac{\sigma_\omega^2}{N}|u_{nk}|^4; \tag{28}$$

$$\langle (E_{nk}^{1,o})^2\rangle = 4\ \mathrm{Re}\left\{\sum_{K,K'} \sigma_{k+K,k+K'}|u_{nk}|^2 v_{nk+K}^* v_{nk+K'}^*\right\}, \tag{29}$$

$$\langle (E_{nk}^{1,p})^2\rangle - \langle E_{nk}^{1,p}\rangle^2 = 4\ \mathrm{Re}\left\{\sum_{K,K'} \zeta_{k+Kk+K'}|u_{nk}|^2 v_{nk+K}^* v_{nk+K'}^*\right\}. \tag{30}$$

The second-order energy shift in the nondegenerate case is given by

$$E_{nk}^{2,e} = \frac{1}{N}\sum_{n'k'} \frac{|\Delta\omega_{k-k'}|^2|u_{nk}|^2|u_{n'k'}|^2}{\Lambda_{nk}-\Lambda_{n'k'}}, \tag{31}$$

$$E_{nk}^{2,o(p)} = \sum_{n'k'} \frac{\left|\sum_K [\eta_{k+Kk'}^{o(p)} u_{n'k'} v_{nk+K}^* + \eta_{k+Kk'}^{*o(p)} u_{nk}^* v_{n'k'+K}^*]\right|^2}{\Lambda_{nk}-\Lambda_{n'k'}}. \tag{32}$$

In the case of energy disorder, average and variance can be calculated as

$$\langle E_{nk}^{2,e}\rangle = \frac{\sigma_\omega^2}{N}\sum_{n'k'} \frac{|u_{nk}|^2|u_{n'k'}|^2}{\Lambda_{nk}-\Lambda_{n'k'}}, \tag{33}$$

$$\langle (E_{nk}^{2,e})^2\rangle - \langle E_{nk}^{2,e}\rangle^2 = \frac{\sigma_\omega^4}{N^2}\sum_{n'k'} \frac{|u_{nk}|^4|u_{n'k'}|^4}{(\Lambda_{nk}-\Lambda_{n'k'})^2}. \tag{34}$$

The average value for the oscillator strength and positional disorder are given by

$$\langle E_{nk}^{2,o(p)}\rangle = \sum_{n'k'} \frac{\chi_{nkn'k'}^{o(p)}}{\Lambda_{nk}-\Lambda_{n'k'}}, \tag{35}$$

where

$$\chi_{nkn'k'}^{o(p)} = \sum_{KK'} [\vartheta_{kk'KK'}^{1o(p)}|u_{n'k'}|^2 v_{nk+K}^* v_{nk+K'}^* + \vartheta_{kk'KK'}^{2o(p)} u_{nk} u_{n'k'} v_{nk+K}^* v_{n'k'+K'}^* + \vartheta_{k'kKK'}^{*2o(p)} u_{nk}^* u_{n'k'}^* v_{nk+K'} v_{n'k'+K} + \vartheta_{k'kKK'}^{1o(p)} \times |u_{nk}|^2 v_{n'k'+K} v_{n'k'+K'}],$$

$$\vartheta_{kk'KK'}^{1o(p)} = \langle \eta_{k+Kk'}^{o(p)} \eta_{k+K'k'}^{*o(p)}\rangle,$$

$$\vartheta_{kk'KK'}^{2o(p)} = \langle \eta_{k+Kk'}^{o(p)} \eta_{k'+K'k}^{o(p)}\rangle. \tag{36}$$

Expressions for the variance are not shown but their calculation is straightforward. The degenerate case for the second-order shift will be discussed in the next section.

III. RESULTS AND DISCUSSION

Here we present the absorption spectra calculated numerically for various strengths of disorder and averaged over many disorder realizations. We consider the case of an excitation at the M point [finite excitation angle corresponding to $q_M=(\pi/a,\pi/a)$] and at the Γ point (normal excitation). The exciton energy is tuned into resonance with the corresponding cavity modes. Therefore, the calculations for Γ and M point refer to different structures with lattice constants a and cavity length L_C adjusted to obtain a strong coupling at the two points (for more details see Table III of Ref. 7). We have used the material parameters of CdSe/ZnSe. The full Hamiltonian corresponding to a lattice with 31^2 sites and photon modes in an energy window $[0.9\omega_X,\ 1.1\omega_X]$ is diagonalized. In order to make the results independent of the system size we keep the coupling constant $\tilde{g}_q$ fixed and we use periodic boundary conditions.

Let us start with the effect of the energy disorder in the absorption spectra as shown in Fig. 2. The disorder-free case (black line) shows lower (LP) and upper (UP) polariton peaks with half-exciton and half-photon character. There is an additional central peak in the M point [Fig. 2(a)]. This peak arises from the mixing of photonic modes that differ by reciprocal lattice vectors. For symmetry reasons one of those mixed states is decoupled from the quantum dots at the M point and we call this peak "photonic." However, this mode couples with the exciton mode away from the M point. The polariton splitting, i.e., the energy difference between the lower and the upper polariton energy, is larger by ~ 0.5 meV at the Γ point than at the M point due to the stronger light matter coupling in the normal direction.[7] The presence of a strong central peak could make the experimental observation of the M point triplet more challenging. The disorder shifts the LP (UP) energy: According to perturbation theory, the shift of the LP (UP) is zero to first order [as seen from Eq. (25)] and is negative (positive) to second order [according to Eq. (33)]. Therefore, we expect an *increase* in the polariton splitting. Weak disorder can increase the robustness and visibility of the polariton splitting with respect to the disorder-free case. The fact that a weak exciton disorder increases the observability of a coherent effect may be seen as counterintuitive, but the same effect has also been found in similar systems (e.g., in the exciton Aharonov-Bohm effect).[23,24] In the case of quantum well polaritons this result can also be obtained by using a coupled oscillator model.[25] We see in Fig. 2 that the UP and LP peaks broaden by increasing the disorder strength. Within the perturbative approach, we find that the broadening of the polariton modes is equal to $\frac{\sigma_\omega}{\sqrt{N}}$ in the lowest order [see Eq. (28)], while the second order is proportional to $\frac{\sigma_\omega^2}{N}$ [see Eq. (34)]. We have also performed a direct comparison between the full numerical solution and the analytical result from perturbation theory for weak disor-

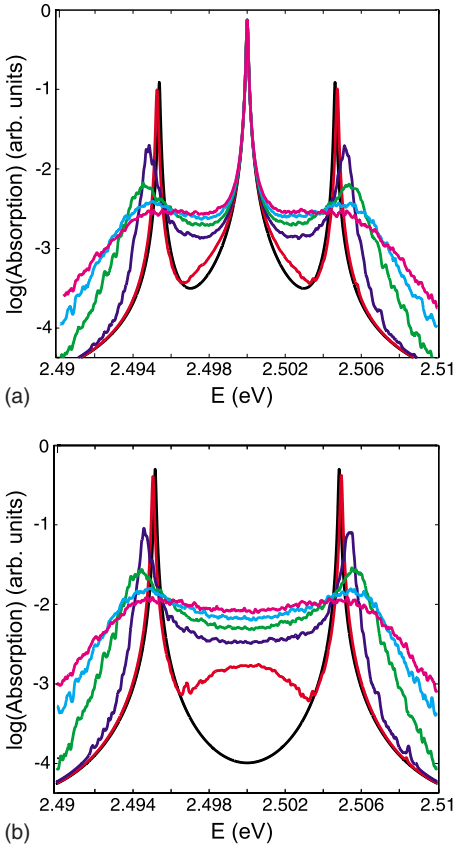

FIG. 2. (Color online) Polariton absorption spectrum in a logarithmic scale for energy disorder with variance from 0 (black, lowest) to 5 meV (magenta, top) in steps of 1 meV for polaritons at two high-symmetry points of the first BZ: (a) M and (b) Γ. A homogeneous Lorentzian broadening of $\gamma=50$ μeV was considered. Material parameters correspond to ZnSe/CdSe systems and can be found in Ref. 7.

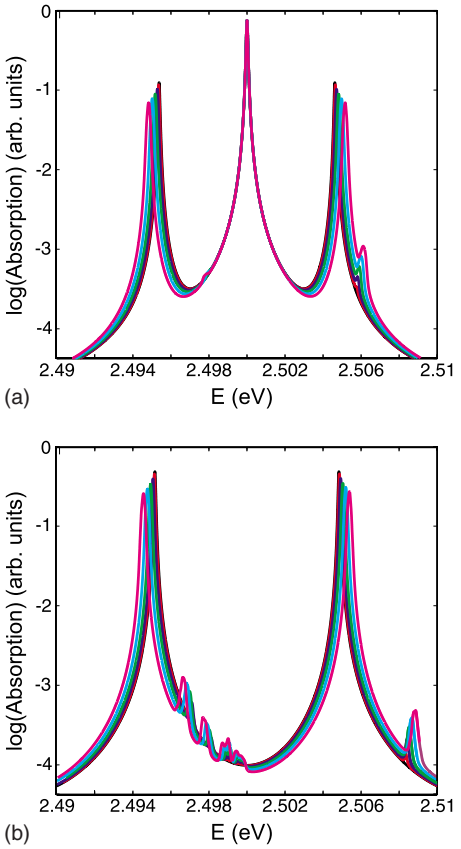

FIG. 4. (Color online) Polariton absorption spectrum in a logarithmic scale for oscillator strength disorder with variance from 0% (black, no side peaks) to 50% (magenta, strongest side peaks) increasing by steps of 10%. Polaritons at two high-symmetry points of the first BZ are shown: (a) M and(b) Γ. Material parameters as in Fig. 2.

der (shown in Fig. 3). The small energy shift is due to the fourth-order and higher-order terms that are not included in the perturbation-theory curve.

Notice that for both Γ and M polaritons, a central very broad excitonic peak appears due to disorder. In the M-point case, however, a central peak insensitive to the disorder is superposed to the broad central excitonic background. As the width of the inhomogeneous energy distribution approaches the value of the lower to upper polariton splitting, the three

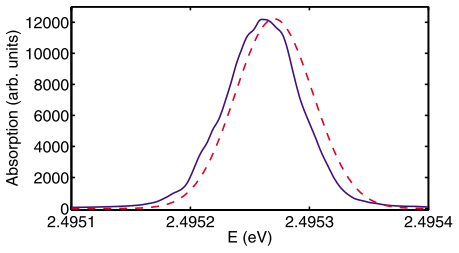

FIG. 3. (Color online) Absorption spectrum in the vicinity of the lower polariton energy calculated exactly (blue) and using the perturbation theory (red, dashed). 2000 realizations of energy disorder with variation $\sigma_\omega=1$ meV on a lattice with 20^2 sites were considered.

peaks merge. In the strong disorder limit the spectrum is consists of a central narrow (photonlike) peak and a broad excitonic background.

From an experimental point of view, state-of-the-art quantum dot samples have inhomogeneous broadening of several millelectron volts, which is comparable to typical values of polariton splitting. This makes the observation of the polariton splitting challenging. On the other hand, impurities have very narrow energy distribution. A planar lattice of impurities can be realized by patterned ion implantation methods. Array of impurities with deep excitonic levels could be obtained by this method. Assuming deep energy levels, Frenkel-like excitons bound to impurities will have a very small inhomogeneous energy broadening. Nevertheless, it is hard to control the number of impurities on every lattice site using ion implantation. The lattice of impurities has thus a very narrow on-site energy distribution but has a large oscillator strength disorder due to the fluctuations in the number of impurities. The same kind of disorder occurs in atoms trapped in optical lattices. In order to investigate this effect, we consider a Gaussian distribution of the exciton oscillator strength in the coupling constants g_{jq}. The absorption spectra for different oscillator strength disorder are shown in Fig. 4. The behavior of the LP and UP peaks for both Γ and M point

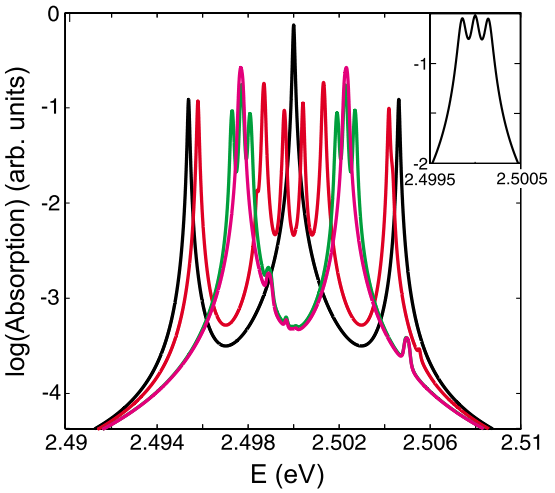

FIG. 5. (Color online) Polariton absorption spectrum in a logarithmic scale for position disorder with variance from 0% (black, no side peaks) to 10% (red), 30% (green), and 50% (magenta, strongest side peaks) of the lattice constant. Polariton modes at the M point of the first BZ are considered. Inset: Detail of the central polariton peak for 1% disorder. Material parameters as in Fig. 2.

is similar to the case of energy disorder. In particular, we observe a decrease (increase) in LP (UP) energy, which is again consistent with the perturbation theory as seen in Eqs. (26) and (35). We also remark that even for very large fluctuations of the oscillator strength both the broadening and energy shift remain smaller with respect to the inhomogeneous energy disorder. The additional peaks seen between and above the two main peaks are caused by polariton multiple scattering, i.e., scattering involving polariton states with different q. Only few peaks are visible because of the finite size of the system in the numerical simulation. The asymmetry reflects the polariton band dispersion. Note that in the case of quantum wells multiple-scattering effects have been shown to be negligible.[26] Here we have an example of a disorder for which the full q dependence of the polaritonic states is important.

Finally, we have also investigated absorption spectra in the presence of positional disorder, i.e., deviations in the exciton localization sites from the ideal lattice (as shown in Fig. 5). Clearly, this disorder plays a role only for M-point polaritons since the Γ point does not probe the lattice symmetry. In contrast to the oscillator strength disorder and to the energy disorder, positional disorder leads to a blueshift (redshift) of the lower (upper) polariton peak, which implies a reduction in the polariton splitting. This is related to the fact that the first-order perturbation theory is nonzero in this case, as seen in Eq. (27). The behavior of the central polariton (CP) peak is more complex. First, it splits into three lines for weak disorder (see inset of Fig. 5). Then the remaining central peak splits again into a doublet for stronger disorder (red curve in Fig. 5) black. The first splitting can be understood taking into account the CPs threefold degeneracy[7] at q_M, which is removed in the presence of disorder. Consequently, one mode is blueshifted, one mode is redshifted, and the third one remains unchanged. By further increasing the disorder strength, the state with q_M mixes with neighboring q and a doublet appears.

For positional disorder some analytical results can be obtained by adding an exciton hopping term between neighboring dots. This removes the degeneracy of the excitonic states in K space, and the second-order degenerate perturbation theory for the three polariton modes at the M point can be applied (the first order is zero). The matrix elements of the second-order effective Hamiltonian read

$$\Theta_{mn}^{o(p)} = \kappa_{mn}^{o(p)} \sum_{n'k'} \frac{|u_{n'k'}|^2}{\Lambda_{CPk_M} - \Lambda_{n'k'}}, \tag{37}$$

where

$$\kappa_{mn}^{o(p)} = \sum_{KK'} \eta_{k+Kk'}^{o(p)} \eta_{k+K'k'}^{*o(p)} v_{mk+K}^{*} v_{nk+K'}. \tag{38}$$

The correction to the unperturbed energy is found by averaging the eigenvalues of Θ, which cannot be done analytically. A good approximation can be obtained by averaging the matrix elements,

$$\langle \kappa_{mn}^{o(p)} \rangle = \vartheta_{kk'KK'}^{1o(p)} v_{mk+K}^{*} v_{nk+K'}, \tag{39}$$

before calculating the eigenvalues. We have shown that this approximation introduces corrections of the fourth order in the disorder potential and gives a good agreement with the exact numerical calculation for small disorder.

If the positional disorder is of the same order as the lattice constant, i.e., for $\sigma_R \sim 0.5a$, the lattice becomes equivalent to random distribution of sites. In this limit, the random phase e^{iqR_j} in the coupling constants can be averaged in the Hamiltonian, which leads to a position independent value $\langle g e^{iqR_j} \rangle = g$. This results in a two-peak spectrum that qualitatively looks like the one in the Γ point resonant system with an upper and lower polariton as clearly seen for the strongest disorder in Fig. 5 (magenta, curve with two main peaks). Such a spectrum with only two peaks is also being obtained in a quantum well microcavity for arbitrary values of the angle and detuning.

In the literature, there is an extensive discussion about the need of a full theoretical treatment of polaritons in a disordered potential versus an approach known as *linear dispersion theory*, a phenomenological model representing all disorder effects through spatially homogeneous excitons with *ad hoc* inhomogeneous energy lines. It is well known that the linear dispersion theory can give a good agreement with the experiment if the characteristics of the disorder and the polariton dispersion are such that polariton multiple scattering does not considerably contribute to the linewidth (see for instance Ref. 26). For the oscillator strength and positional disorder studied here, it is not possible to introduce an effective linear dispersion theory approach since there is no energy broadening of the exciton line. For the energy disorder at the Γ and M point we have introduced *one* symmetric photon mode, which couples to the exciton [see Eq. (11) and corresponding discussion in Ref. 7] and we have verified that the linear dispersion theory gives absorption spectra qualitatively similar to the ones calculated numerically.

IV. CONCLUSIONS

We have investigated numerically and analytically the role of disorder in structures consisting of a lattice of dots or impurities embedded in a planar microcavity. We have focused on Bragg polaritons occurring at the Brillouin zone boundary and we have shown that small energy disorder is necessary to observe polariton features in the absorption spectra. Moreover, we have found that the Bragg polariton resonance is very robust against oscillator strength and position disorder. Our results show that a lattice consisting of deep impurities with Frenkel excitons or atoms in an optical lattice structure are promising candidates for the experimental observation of Bragg polaritons.

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